# Two-Phase Mach Numbers in Heat Pipe Analysis

JOSEPH W. BURSIK*
*Rensselaer Polytechnic Institute, Troy, N.Y.*

## Theme

IN analyzing one-dimensional flow of a compressible two-phase mixture in a heat pipe of constant vapor passage diameter $D$, Levy[1] uses several plausible assumptions to obtain approximate digital integration of his equations. His solutions suggest without explicit proof infinite gradients in various properties at the downstream end of the evaporator. In this paper the gradients are re-examined after transforming them in terms of an equilibrium two-phase Mach number. It is then shown that the assumption of a uniform mass injection rate $\dot{m}_e = \pi D \rho_g V_n$, where $V_n$ is the radial mass injection velocity, apparently leads to infinite gradients; however, second law analysis rules out a constant $\dot{m}_e$ as well as a uniform heat addition rate per unit length of evaporator, $Q_e = \dot{m}_e(h_{fg} + V_n^2/2)$, leaving the gradients indeterminate. The subscripts $f$ and $g$ denote liquid and vapor phases.

## Contents

The equilibrium two-phase Mach number is obtained from well known thermodynamic equations specialized for the two-phase region.[2,3] The first is

$$T ds = \sigma_v dt + Tp'(T) dv \tag{1}$$

and this results from a one-phase form whose $c_v$ has been replaced by a two-phase heat capacity $\sigma_v$. In addition, the vapor pressure curve slope has replaced the one-phase $(\partial p/\partial T)_v$ in the coefficient of $dv$. With $dp = p'(T)\,dT$ and the equilibrium sound speed given by $-v^2(\partial p/\partial v)_s$, Eq (1) gives a two-phase sound speed as

$$a^2 = Tv^2 p'(T)^2/\sigma_v \tag{2}$$

Also from thermodynamics, the differential Gibbs' function is $dg = vdp - sdT$. For a two-phase region of a pure substance $dg = g'(T)dT$ and this combines with the latter to give $s = vp'(T) - g'(T)$, from which

$$ds = [vp''(T) - g''(T)]dT + p'(T)dv \tag{3}$$

Double primes denote second derivatives. Comparison with Eq. (1) gives

$$\sigma_v(T,v) = T[vp''(T) - g''(T)] \tag{4}$$

Equations (2) and (4) give the two-phase sound speed as a function of $T$ and $v$, and the two-phase Mach number is defined in terms of the axial velocity $V$ as $M = V/a$.

Using sodium properties,[4] the two-phase Mach number $M_{gII}$ of a mixture possessing an infinitesimal amount of liquid and an axial velocity $V$ is compared to a monatomic one-phase Mach number $M_{gI}$ of an infinitesimally superheated ideal gas moving with the same velocity. At 590°F an approximate computation neglecting dimerization and using finite differences for derivatives gives $M_{gII}/M_{gI} = 1.41$. At 1100°F the ratio is 1.28, with a monotonic trend at intermediate temperatures.[3] This illustrates significant differences resulting in ideal gas vs two-phase theory.

The basic inviscid, one-dimensional equations of Levy are transformed in Ref. 3 to contain the two-phase Mach number, an additional quantity defined as $e^2 = h_g - h + (V^2 + V_n^2)/2$ and $\mu$ defined as the ratio $\dot{m}_e/\dot{m}$, where $\dot{m} = \rho a v$ is the axial mass flow rate of the two-phase mixture. With $z$ as the axial dimension of the evaporator, some resulting gradient forms are

$$dp/dz = -\mu p' M^2(e^2 + 2Tvp')/\sigma_v(1 - M^2) \tag{5}$$

$$dV/dz = \mu V[\{(e^2 + 2Tvp')/Tvp'(1 - M^2)\} - 1] \tag{6}$$

and

$$ds/dz = \mu(e^2/T) \tag{7}$$

From the second law, $ds \geqq 0$ and from Eq. (7) $e^2$ must therefore be positive. Reference 3 also contains the gradients in the specific volume and enthalpy of the two-phase mixture. For $M < 1$, the pressure gradient is negative, the velocity gradient is positive as is the entropy gradient. Since $dp = p'(T)dT$, Eq. (5) shows that the temperature gradient is also negative.

If $\dot{m}_e \neq 0$ at the two-phase $M = 1$ section the gradients in pressure, temperature, velocity, specific volume and enthalpy are all infinite.

Finally, there are numerous references in the literature describing the use of an adiabatic section between the evaporator and the condenser with choking taking place at this section.[5-7] This adiabatic choking condition is now investigated. When $dp$ in Eq. (5) is replaced by $p'(T)dT$ and the result is combined with Eq. (7) an expression is obtained for $dT/ds$ that is independent of $\mu$ and inversely proportional to $1 - M^2$, such that when $M = 1$, $dT/ds$ becomes infinite. That is $ds$ must be zero at $M = 1$. This suggests a curve in the $s$, $Z$ plane in which the entropy increases with $Z$ until it reaches a maximum at a section where $M = 1$, such that $ds/dZ$ at this section is zero. This requires the right side of Eq. (7) to be zero. For finite axial flow rates $\dot{m}_e$ must be locally zero at $M = 1$ and this requires both $V_n$ and $Q_e$ to be locally zero at $M = 1$, thereby requiring variable $Q_e$ and $\dot{m}_e$. By setting the mass injection term locally zero at $M = 1$, the two-phase flow is not only adiabatic but the mixing of the radial flow with the axial momentarily ceases, permitting local isentropic conditions to prevail at $M = 1$. The various gradients then become indeterminate at $M = 1$. Thus, the discharge end of the evaporator cannot be identified as the Mach 1 section.

## References

[1] Levy, E. K., "Theoretical Investigation of Heat Pipes Operating at Low Vapor Pressures," *ASME Journal of Engineering for Industry*, Nov. 1968, p. 547.

[2] Bursik, J. W., "Pseudo-Sonic Velocity and Pseudo-Mach Number Concepts in Two-Phase Flows," TN D-3734, Dec. 1966, NASA.

[3] Bursik, J. W., "The Role of Two-Phase Mach Numbers in Heat Pipe Analysis," AIAA Paper 72-22, San Diego, Calif., 1972.

[4] Golden, G. H., Tokar, J. V., and Miller, D., "Thermophysical Properties of Sodium—Recommended Values," *Reactor and Fuel Processing Technology*, Vol. 11, No. 1, Winter 1967–1968, pp. 27–44.

[5] Kemme, J. E., "Ultimate Heat Pipe Performance," *IEEE Transactions on Electron Devices*, Vol. ED-16, No. 8, Aug. 1969, pp. 712–723.

[6] Deverall, J. E., Kemme, J. E., and Florschuetz, L. W., "Sonic Limitations and Startup Problems of Heat Pipes," LA-4518, Nov. 1970, Los Alamos Scientific Lab., Univ. of California, Los Alamos, N. Mex.

[7] Dzakowic, G. S., Tang, Y. S., and Arcella, F. G., Experimental Study of Vapor Velocity Limit in Sodium Heat Pipe," ASME paper 69 HT 21, *ASME-AIChE Heat Transfer Conference*, Minneapolis, Minn., Aug. 1969.

Presented as Paper 72-22 at the AIAA 10th Aerospace Sciences Meeting, San Diego, Calif., January 17–19, 1972; submitted July 5, 1972; synoptic received September 28, 1972. Full paper available from AIAA Library, 750 Third Avenue, New York, N.Y. 10017. Price: Microfiche, $1.00; hard copy, $5.00. **Order must be accompanied by remittance.**

Index category: Fluid Dynamics.

* Associate Professor of Aeronautical Engineering.